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Production Inventory Models for Deteriorating Items with Stochastic Machine Unavailability Time, Lost Sales and Price-Dependent Demand

Gede Agus Widyadana[1], Hui Ming Wee[2]

Abstract: The economic production quantity (EPQ) model is widely employed in reality and is also being intensively developed in the research area. This research tries to develop more realistic EPQ models for deteriorating items by considering stochastic machine unavailability time (uniformly and exponentially distributed) and price-dependent demand. Lost sales will occur when machine unavailability time is longer than the non production time. Since the closed form solution cannot be derived, we use Genetic Algorithm (GA) to solve the models. A numerical example and sensitivity analysis is shown to illustrate the models. The sensitivity analyses show that a management can use price policy to minimize the profit loss due to machine unavailability time under a price- dependent demand situation.

Keywords: EPQ, deteriorating items, machine unavailability time, price-dependent demand.

Introduction

Production inventory models have been intensively investigated and most of the researches try to fit the models to real-world inventory problems. In recent years, many researchers had been trying to develop EPQ models for unreliable production facility since it is difficult to set a reliable facility in reality. Some production facilities face problems on producing the products on the right time and sometimes even defective products are produced. Unreliable production facility might be caused by material unavailability, machine repair or maintenance, and machine breakdown.

Abboud *et al.* [1] developed EPQ models for unreliable machine where machine is not available when needed due to some problems such as secondary jobs being processed, materials were unavailable, or the machine is maintained. They assumed the machine unavailability time is uniformly and exponentially distributed. Giri and Dohi [7] developed EPQ models where machine capacity can be determined before the production run and the corrective and preventive maintenance times follow a general distribution. An EPQ model with stochastic machine breakdown, scrap products, and rework process was considered by Chiu *et al.* [4]. El-Ferik [6] developed an EPQ model under preventive maintenance schedule where the production facility is subject to random failure and the maintenance process is imperfect.

Some items such as electronics, fruits and milks are deteriorating through time. Deterioration is defined as decay, damage, spoilage, evaporation, obsolescence, pilferage, and loss of entity or marginal value of a commodity that results in decreased usefulness from the original one (Wee [15]). The EPQ model for deteriorating items was initially developed by Misra [9]. Alfares *et al.* [2] developed production-inventory models for both deteriorating items and deterioration process. They also considered some realistic aspects in their models such as varying demand and production rates, quality, inspection and maintenance. A production-inventory model for deteriorating items with imperfect production process was developed by Lin and Gong [8]. Chung *et al.* [5] extended the work of Abboud *et al.* [1] by developing EPQ models for deteriorating items and considered random machine unavailability, lost sales and backorder.

Some demands are price-dependent, therefore price is considered as an important decision to determine the maximum total profit. Wee and Law [16] analyzed a deteriorating inventory model by considering price-dependent demand and time value of money. Teng and Chang [13] developed an EPQ model for deteriorating items with price- and stock-dependent demand. They assumed that the demand rate depends on the price and the number of on-display stocks. Chen and Chen [13] developed an

[1] Faculty of Industrial Technology, Department of Industrial Engineering, Petra Christian University. Jl. Siwalankerto 121-131, Surabaya 60236, Indonesia, Email: gede@petra.ac.id,
[1,2] Faculty of Technology, Department of Industrial and System Engineering, Chung Yuan Christian University, Chung Li 32023, Taiwan, Email: weehm@cycu.edu.tw

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inventory model for deteriorating items subject to price-dependent and time varying demand, time varying deterioration rate, production rate, and variable production cost. A price- dependent demand in a deteriorating inventory model with markdown price option was considered by Widyadana and Wee [17]. Tsao and Sheen [14] developed the supplier's trade credit and retailer's promotional effort in finite horizon inventory model for deteriorating items with price- and time-dependent demand environment.

According to our extensive literature study above, there is no research that focuses on developing EPQ models for deteriorating items with stochastic machine unavailability time and price-dependent demand. This paper extends Chung *et al.* [5] model's by considering price-dependent demand. A possibility arises for some manufacturers to carefully determine their price by considering their production facility conditions. The EPQ models assume that machine unavailability time is uniformly and exponentially distributed. There are two decision variables of interest which are the optimal production uptime and the optimal product price. We use the Genetic Algorithm (GA) to solve the problem since the closed-form solution cannot be derived. Genetic algorithm is one of the heuristic methods that mimic the natural evolution process and has been used widely to solve NP-hard problems. The use of GA to solve inventory problems can be found in Mondal and Maiti [10], Pourakbar *et al.* [12], and Pasandideh and Niaki [11].

This paper is organized as follows. The research motivation and the literature review are presented in Section 1. The mathematical model development and the GA method used to solve the model are discussed in Section 2. Result and discussion are presented in Section 3 with the sensitivity analyses to illustrate the developed models and provide some managerial insight. Conclusions and future research are given in the last section.

Methods

The following assumptions are used in the model development: (1) Production is constant. (2) Deterioration rate is constant. (3) There is no repair or replacement for a deteriorated item. (4) Demand rate depends on price. The demand rate equation is $\alpha p^{-\varepsilon}$.

The following notations are used in the model:

I : inventory level (unit)
I_1 : inventory level in production period (unit)
I_2 : inventory level in non-production period (unit)
T : replenishment period
T_1 : production period
T_2 : non-production period
T_3 : lost sales period
P : production rate (unit/time)
θ : deterioration rate
α : constant price dependent parameter
ε : increased price rate
p : product price ($)
K : setup cost ($/setup)
h : holding cost ($/unit/time)
S : lost sales cost ($/unit)
π : deterioration cost ($/unit/time)
TP : total profit
TPT : total profit per unit time
TPT_{NL} : total profit per unit time when lost sales do not occur
TPT_U : total profit per unit time for uniform distribution case
TPT_E : total profit per unit time for exponential distribution case

The inventory policy for lost sales case is illustrated in Figure 1. Production is performed during T_1 time period. When inventory reach maximum level I_m, the production stops and the inventory decreases due to demand and deterioration. The inventory level reaches zero units at time (T_1 +T_2), and the machine is started again to produce the item. Since machine unavailability is randomly distributed that has a probability density function *f*(*t*), production may be impossible to start. If machine is unavailable, lost sales occur during T_3 time period. The production system is then run after T_3 time period.

The inventory level in a production period from the above problem can be formulated as:

$$\frac{dI_1(t_1)}{dt_1} + \theta I_1(t_1) = P - \alpha p^{-\varepsilon} \quad 0 \le t_1 \le T_1 \tag{1}$$

While the number of inventory level in a non production period is modeled as follows:

$$\frac{dI_2(t_2)}{dt_2} + \theta I_2(t_2) = -\alpha p^{-\varepsilon} \quad 0 \le t_2 \le T_2 \tag{2}$$

Since $I_1(0) = 0$, and $I_2(T_2) = 0$, the inventory level in a production and a non production period respectively are:

$$I_1(t_2) = \frac{\alpha p^{-\varepsilon}}{\theta}\left(e^{-\theta(T_2-t_2)} - 1\right) \quad 0 \le t_2 \le T_2$$

$$I_1(t_1) = \frac{P-\alpha p^{-\varepsilon}}{\theta}\left(1 - e^{-\theta t_1}\right) \quad 0 \le t_1 \le T_1 \tag{3}$$

$$I_2(t_2) = \frac{\alpha p^{-\varepsilon}}{\theta}\left(e^{\theta(T_2-t_2)} - 1\right) \quad 0 \le t_2 \le T_2 \tag{4}$$

Since $I_1 = I_2$ when $t_1 = T_1$ and $t_2 = 0$ then one has:

$$\frac{P-\alpha p^{-\varepsilon}}{\theta}\left(1 - e^{-\theta T_1}\right) = \frac{\alpha p^{-\varepsilon}}{\theta}\left(e^{\theta T_2} - 1\right) \tag{5}$$

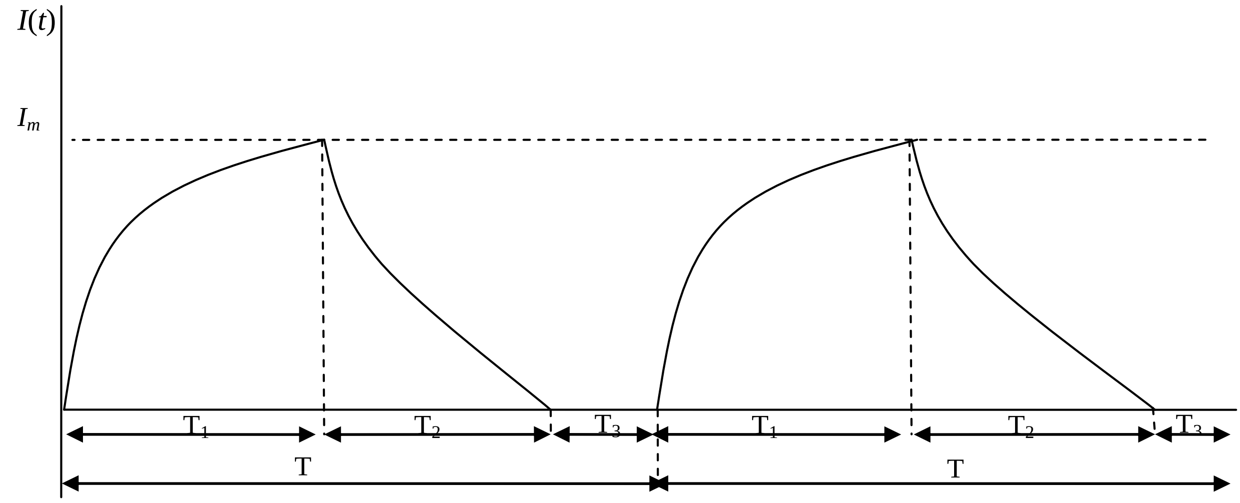

Figure 1. Inventory level of lost sales case

Using the Taylor series approximation (Yang and Wee [18]), then (5) can be modeled as follows:

$$(P-\alpha p^{-\varepsilon})(T_1-\tfrac{1}{2}\theta T_1^2) \;=\; \alpha p^{-\varepsilon}(T_2+\tfrac{1}{2}\theta T_2^2) \qquad (6)$$

Since T_2 is a small number, T_2 in terms of T_1 can be approximated as:

$$T_2 \cong \frac{(P-\alpha p^{-\varepsilon})T_1(1-\frac{1}{2}\theta T_1)}{\alpha p^{-\varepsilon}} \qquad (7)$$

This approximation is commonly used in deteriorating production inventory problem (see Misra, [9]). The expected inventory level is:

$$E(I) \;=\int_{t_1=0}^{T_1}\frac{P-\alpha p^{-\varepsilon}}{\theta}\left(1-e^{-\theta t_1}\right)dt_1 + \int_{t_2=0}^{T_2}\frac{\alpha p^{-\varepsilon}}{\theta}\left(e^{\theta(T_2-t_2)}-1\right)dt_2$$

$$E(I)=\frac{P-\alpha p^{-\varepsilon}}{\theta^2}\left(\theta T_1+e^{-\theta T_1}-1\right) + \frac{\alpha p^{-\varepsilon}}{\theta^2}\left(-\theta T_2+e^{\theta T_2}-1\right) \qquad (8)$$

Using Taylor series approximation, (8) can be remodeled as follows:

$$E(I)=(P-\alpha p^{-\varepsilon})\left(\frac{T_1^2}{2}\right)+\alpha p^{-\varepsilon}\left(\frac{T_2^2}{2}\right) \qquad (9)$$

Substituting (7) to (9), one has:

$$E(I)=(P-\alpha p^{-\varepsilon})\left(\frac{T_1^2}{2}\right) + \frac{\alpha p^{-\varepsilon}}{2}\left(\frac{(P-\alpha p^{-\varepsilon})T_1(1-\frac{1}{2}\theta T_1)}{\alpha p^{-\varepsilon}}\right)^2 \qquad (10)$$

Since $\frac{1}{2}\,\theta T_1$ is very small, (10) can be simplified as:

$$\mathrm{E(I)} \cong \frac{\mathrm{P}^2}{2\alpha \mathrm{p}^{-\varepsilon}}\left(1-\frac{\alpha \mathrm{p}^{-\varepsilon}}{\mathrm{P}}\right)\mathrm{T}_1^2 \qquad (11)$$

The expected deteriorated items are equal to the total production items minus the total demand, and can be formulated as:

$$\mathrm{E(R)}=\mathrm{PT}_1-\alpha\mathrm{p}^{-\varepsilon}(\mathrm{T}_1+\mathrm{T}_2) \qquad (12)$$

Substituting T_2 from (7) to (12), then (12) can be simplified as:

$$\mathrm{E(R)} \cong \frac{\mathrm{P}}{2}\left(1-\frac{\alpha\mathrm{p}^{-\varepsilon}}{\mathrm{P}}\right)\theta\mathrm{T}_1^2 \qquad (13)$$

The production cost can be formulated as:

$$\mathrm{TC_p}=\mathrm{C_pPT_1} \qquad (14)$$

The total revenue is equal to demand in the production up time and the down time period and it can be modeled as:

$$\mathrm{V}=\alpha\mathrm{p}^{1-\varepsilon}(\mathrm{T}_1+\mathrm{T}_2) \qquad (15)$$

The total profit consists of total revenue minus setup cost, production cost, holding cost, deteriorating cost and lost sales cost. The total profit can then be expressed as:

$$TP(T_1,T_2,p)=\alpha p^{1-\varepsilon}(T_1+T_2) - \left(\mathrm{K}+\mathrm{C_pPT_1}+\left(\frac{\mathrm{hP}}{\alpha\mathrm{p}^{-\varepsilon}}+\pi\theta\right)\frac{\mathrm{PT}_1^2}{2}\left(1-\frac{\alpha\mathrm{p}^{-\varepsilon}}{\mathrm{P}}\right)+ \mathrm{S}\alpha\mathrm{p}^{-\varepsilon}\int_{\mathrm{t=T_2}}^{\infty}(\mathrm{t}-\mathrm{T}_2)\mathrm{f(t)dt}\right) \qquad (16)$$

The total replenishment time is equal to the production up time period, the non production period and the machine unavailable time probability. The expected total replenishment time can be formulated as:

$$E(T) = T_1 + T_2 + \int_{t=T_2}^{\infty}(t - T_2)f(t)dt \quad (17)$$

Using the renewal reward theorem, the expected total profit per unit time can be formulated as follows:

$$TPT(T_1, T_2, p) = \frac{\alpha p^{1-\varepsilon}(T_1+T_2)-(K+C_pPT_1)}{T_1+T_2+\int_{t=T_2}^{\infty}(t-T_2)f(t)dt}$$

$$-\frac{\left(\left(\frac{hP}{\alpha p^{-\varepsilon}}+\pi\theta\right)\frac{PT_1^2}{2}\left(1-\frac{\alpha p^{-\varepsilon}}{P}\right)+S\alpha p^{-\varepsilon}\int_{t=T_2}^{\infty}(t-T_2)f(t)dt\right)}{T_1+T_2+\int_{t=T_2}^{\infty}(t-T_2)f(t)dt} \quad (18)$$

Uniform Distribution Case

It is assumed that the machine repair time *t*, is a random variable that is uniformly distributed over the interval [0, b]. The probability density function, *f*(*t*), is given as:

$$f(t) = \begin{cases} 1/b, & 0 \le t \le b \\ 0, & \text{otherwise} \end{cases} \quad (19)$$

The expected shortage time can be written as:

$$E(T_3) = \frac{1}{b}\int_{t=T_2}^{b}(t - T_2)\,dt$$

$$E(T_3) = \frac{\left(b-\frac{(P-\alpha p^{-\varepsilon})T_1(1-\frac{1}{2}\theta T_1)}{\alpha p^{-\varepsilon}}\right)^2}{2b} \quad (20)$$

Simplifying the equation above, one has:

$$E(T_3) = \frac{\left(b-\frac{(P-\alpha p^{-\varepsilon})T_1}{\alpha p^{-\varepsilon}}\right)^2}{2b} \quad (21)$$

Substituting (7) and (21) to (18), the total cost per unit time of uniform distribution repair time is in equation (22).

$$TPT_U(T_1,p) = \frac{\alpha p^{1-\varepsilon}\left(T_1+\frac{(P-\alpha p^{-\varepsilon})T_1(1-\frac{1}{2}\theta T_1)}{\alpha p^{-\varepsilon}}\right)-(K+C_pPT_1)}{T_1+\frac{(P-\alpha p^{-\varepsilon})T_1(1-\frac{1}{2}\theta T_1)}{\alpha p^{-\varepsilon}}+\frac{\left(b-\frac{(P-\alpha p^{-\varepsilon})T_1}{\alpha p^{-\varepsilon}}\right)^2}{2b}}$$

$$-\frac{\left(\frac{hP}{\alpha p^{-\varepsilon}}+\pi\theta\right)\frac{PT_1^2}{2}\left(1-\frac{\alpha p^{-\varepsilon}}{P}\right)}{T_1+\frac{(P-\alpha p^{-\varepsilon})T_1(1-\frac{1}{2}\theta T_1)}{\alpha p^{-\varepsilon}}+\frac{\left(b-\frac{(P-\alpha p^{-\varepsilon})T_1}{\alpha p^{-\varepsilon}}\right)^2}{2b}}$$

$$-\frac{\left(\frac{S\alpha p^{-\varepsilon}\left(b-\frac{(P-\alpha p^{-\varepsilon})T_1}{\alpha p^{-\varepsilon}}\right)^2}{2b}\right)}{T_1+\frac{(P-\alpha p^{-\varepsilon})T_1(1-\frac{1}{2}\theta T_1)}{\alpha p^{-\varepsilon}}+\frac{\left(b-\frac{(P-\alpha p^{-\varepsilon})T_1}{\alpha p^{-\varepsilon}}\right)^2}{2b}} \quad (22)$$

If the non production time (T_2) is longer than the upper bound of uniform distribution parameter (*b*), lost sales will not occur and (22) is not valid. Moreover, to handle this situation, the equation (23) below is used:

$$TPT_{NL}(T_1,p) = \frac{\alpha p^{1-\varepsilon}\left(T_1+\frac{(P-\alpha p^{-\varepsilon})T_1(1-\frac{1}{2}\theta T_1)}{\alpha p^{-\varepsilon}}\right)}{T_1+\frac{(P-\alpha p^{-\varepsilon})T_1(1-\frac{1}{2}\theta T_1)}{\alpha p^{-\varepsilon}}}$$

$$-\frac{\left(K+C_pPT_1+\left(\frac{hP}{\alpha p^{-\varepsilon}}+\pi\theta\right)\frac{PT_1^2}{2}\left(1-\frac{\alpha p^{-\varepsilon}}{P}\right)\right)}{T_1+\frac{(P-\alpha p^{-\varepsilon})T_1(1-\frac{1}{2}\theta T_1)}{\alpha p^{-\varepsilon}}} \quad (23)$$

Exponential Distribution Case

In the second case, the machine repair time is a random variable that is exponentially distributed. Exponential probability density function with mean $\frac{1}{\lambda}$ is given as: $f(t) = \lambda e^{-\lambda t} \quad for\ \lambda > 0$. The expected shortage period is:

$$E(T_3) = \int_{t=T_2}^{b}\lambda e^{-\lambda t}\quad dt$$

$$E(T_3) = \frac{e^{-\lambda T_2}}{\lambda} \quad (24)$$

The total cost for exponential distribution case can be formulated by substituting (24) into (18), and one has equation (25).

$$TPT_E(T_1,p) = \frac{\alpha p^{1-\varepsilon}\left(T_1+\frac{(P-\alpha p^{-\varepsilon})T_1(1-\frac{1}{2}\theta T_1)}{\alpha p^{-\varepsilon}}\right)}{T_1+\frac{(P-\alpha p^{-\varepsilon})T_1(1-\frac{1}{2}\theta T_1)}{\alpha p^{-\varepsilon}}+\frac{e^{-\lambda\left(\frac{(P-\alpha p^{-\varepsilon})T_1(1-\frac{1}{2}\theta T_1)}{\alpha p^{-\varepsilon}}\right)}}{\lambda}}$$

$$-\frac{K+C_pPT_1+\left(\frac{hP}{\alpha p^{-\varepsilon}}+\pi\theta\right)\frac{PT_1^2}{2}\left(1-\frac{\alpha p^{-\varepsilon}}{P}\right)}{T_1+\frac{(P-\alpha p^{-\varepsilon})T_1(1-\frac{1}{2}\theta T_1)}{\alpha p^{-\varepsilon}}+\frac{e^{-\lambda\left(\frac{(P-\alpha p^{-\varepsilon})T_1(1-\frac{1}{2}\theta T_1)}{\alpha p^{-\varepsilon}}\right)}}{\lambda}}$$

$$-\frac{\frac{S\alpha p^{-\varepsilon}e^{-\lambda\left(\frac{(P-\alpha p^{-\varepsilon})T_1(1-\frac{1}{2}\theta T_1)}{\alpha p^{-\varepsilon}}\right)}}{\lambda}}{T_1+\frac{(P-\alpha p^{-\varepsilon})T_1(1-\frac{1}{2}\theta T_1)}{\alpha p^{-\varepsilon}}+\frac{e^{-\lambda\left(\frac{(P-\alpha p^{-\varepsilon})T_1(1-\frac{1}{2}\theta T_1)}{\alpha p^{-\varepsilon}}\right)}}{\lambda}} \quad (25)$$

Genetic Algorithm

The Genetic Algorithm method is used to solve the two parameters: production uptime (T_1) and product price (*p*). The method is developed as follows:

Chromosome

The chromosome (allele) is composed of binary digits

$$\overbrace{\boxed{0}\,\boxed{1}\,\boxed{1}\,\boxed{0}}^{T_1}\;\overbrace{\boxed{0}\,\boxed{1}\,\boxed{0}\,\boxed{1}}^{p}$$

Figure 2. Chromosome structure for two parameters

0 and 1 and this is used to represent the production uptime (T_1) and the product price (p). The chromosome structure for the two parameters is shown in Figure (2). Moreover, for example, the T_1 parameter can be represented as T_1 = 0x2^0+1x2^1+1x2^2+0x2^3 = 5/15 = 0.33, the p parameter can be represented as p = 0x2^0+1x2^1+0x2^2+1x2^3 = 10. The numbers of allele depend on the detail of solutions and the problem. In our example we use maximum T_1 = 2, where the total number of allele is equal to 255 and maximum p = 127.

Initial population

The initial chromosome population is generated randomly and the population size is equal to 20. The initial chromosome consists of variables for the production up time (T_1) and the product price (p).

Evaluation of Fitness

A fitness function is calculated using the maximum profit for the uniform distribution case and exponential distribution case.

Parent Selection

This model uses the roulette wheel method with lot size equal to their fitness for reproduction. Since the objective is to maximize the profit, a chromosome with bigger fitness value has a greater probability of being selected.

Genetic Operators

Genetic operators are used to derive better solutions for each generation. Genetic operators consist of elitism, crossover, and mutation. In this study, the population size is set at constant through successive generation. Elitism is a procedure to copy the best chromosome to the next generation. For this problem, two slots are reserved in the next generation for two best chromosomes. In each generation, the elitism is set and the crossover and mutation are used to generate new children.

Crossover

A two point crossover function with a probability of 0.8 was used. In the two point crossover, two positions are selected randomly from parent chromosomes and every allele between the two points of the parents is swapped. An example of a two point crossover is shown below. If we have two parents shown as:

Parent 1

1	2	3	4	5	6	7	8
1	1	1	0	0	1	1	1

Parent 2

1	2	3	4	5	6	7	8
1	0	1	0	1	0	1	0

with the two point crossover at positions 2 and 4, the result would be:

Child 1

1	2	3	4	5	6	7	8
1	0	1	0	0	1	1	1

Child 2

1	2	3	4	5	6	7	8
1	1	1	0	1	0	1	0

Mutation

The mutation scheme is uniform with a mutation probability of 0.05. In this scheme, a parent is randomly selected and one position from this parent is randomly chosen where the value of the allele is changed. An example of the mutation scheme is shown below. If we have one parent shown as:

Parent

1	2	3	4	5	6	7	8
0	0	1	1	0	0	1	1

with the mutation position at the fourth, the value of fourth position is switched from 1 to 0, and the new resulting child would be:

Child

1	2	3	4	5	6	7	8
0	0	1	0	0	0	1	1

Stopping Criterion

The stopping criterion for the GA depends on the predefined number of generations. If the number of generations is greater than the predefined value, then the calculation procedure would stop and the best solution is selected. In this research, we use 100 generations. In the next section, a numerical example is given to illustrate the theoretical results.

Result and Discussion

The following values are considered for the numerical example and the values are set as: K= $

50 per production cycle, P = 1000 unit/time, α = 100000, ε = 1.5, h = $ 1 per unit per unit time, C_p = $ 25 per unit, S = $ 5 per unit, θ = 0.05, unavailability time is uniformly distributed over the interval [0, 1], and π = $ 1 per unit per unit time. Matlab version 7.4 is used to solve the problem. The minimum value of the product price is set as equal to the product cost and the maximum value of the product price is 152. The minimum and maximum production uptime is set to 0 and 2 respectively. In each computation, the GA is run five times and the best solution is chosen. The Genetic Algorithm solution results to T_1 = 0.173, p = 77 and the optimal profit (*TPT*) is $ 7564.816. In this study, the effects of different increased price rate, machine unavailability time parameter, and lost sales cost to the optimal production up time, the optimal price, and the optimal profit are analyzed.

The sensitivity analysis is conducted by changing one parameter by -20%, -10%, +10% and +20% and the remaining parameters are kept unchanged. The sensitivity analysis is carried out for different increased price rate, machine unavailability time, and lost sales cost. The different increased price rate values are chosen as it affects the price decision and the profit. Also, the different values for machine unavailability times and lost sales costs are chosen as it affects the production uptime as well as the profit. The sensitivity analysis for the production up time period (T_1) is shown in Table 1. The table shows that as T_1 increases, the machine unavailability time also increases but is decreasing as the increased price rate is increased. The optimal production time is insensitive in varying values of the lost sales cost. The sensitivity analysis shows that the management should focus more on machine unavailability times than the lost sales cost as it affects more on deriving the optimal production uptime.

Table 1. Sensitivity analysis of T_1 for uniform distribution unavailability time

Parameters	T_1			
	-20% changed	-10% changed	10% changed	20% changed
ε	0.314961	0.283465	0.173228	0.125984
b	0.141732	0.157480	0.173228	0.188976
S	0.173228	0.173228	0.173228	0.173228

Table 2. Sensitivity analysis of p for uniform distribution unavailability time

Parameters	P			
	-20% changed	-10% changed	10% changed	20% changed
ε	152	96	77	64
b	76	76	77	77
S	77	77	77	77

Table 3. Sensitivity analysis of *TPT* for uniform distribution unavailability time

Parameters	*TPT*			
	-20% changed	-10% changed	10% changed	20% changed
ε	30393.784	14788.274	7564.816	3976.039
b	7571.862	7568.296	7564.816	7559.906
S	7564.817	7564.816	7564.816	7564.815

product price tends to decrease as the increased price rate is increasing but the price is insensitive in different values of machine unavailability time and lost sales costs. The product price tends to increase as the machine unavailability time is increasing, however the product price does not increase significantly.

The sensitivity analysis of the total profit is shown in Table 3. This table shows that the total profit tends to decrease as the parameters of increased price rate, machine unavailability time, and lost sales costs are increasing. The total profit does not decrease significantly with increasing lost sales cost. Rather, the former significantly decreases at increased machine unavailability time.

The sensitivity analyses provide managerial insights that can be useful for the management's decision making. The results show that the machine unavailability time affects the total profit. The total profit decreases as the machine unavailability time increases, however management can reduce this loss by increasing the product price. The increased price rate has the most significant effect to the total profit compared to machine unavailability time and lost sales cost. Still, the increased price rate is the most difficult part to be managed among all other parameters.

Conclusion

A production inventory models with stochastic machine unavailability time (uniformly and exponentially distributed unavailability time) have been developed. The sensitivity analyses show that the price rate is the most sensitive parameter than the machine unavailability time and the lost sales cost. The price rate, however, is the most difficult parameter to be managed. The machine unavailability time also affects the total profit as this decreases when the machine unavailability time increases. However, the management can reduce the loss by increasing the product price. This shows that in price-dependent demand situation, the management can use price policy to minimize the loss in profit due to machine unavailability time. But this still may reduce the management's profit. In this paper, it is assumed that machine unavailability time occurs after the production uptime has been completed. In reality, there is a possibility that the machine breakdown will occur before the predetermined production uptime and it is believed as interesting situation to be considered for future research.

References

1. Abboud, N. E., Jaber, M. Y., and Noueihed, N. A., Economic Lot Sizing with the Consideration of Random Machine Unavailability Time, *Computers & Operations Research*, 27, 2000, pp. 335 – 351.
2. Alfares, H. K., Khursheed, S. N., and Noma, S. M., Integrating Quality and Maintenance Decisions in a Production-Inventory Model for Deteriorating Items, *International Journal of Production Research*, 43, 2005, pp. 899 – 911.
3. Chen, J. M., and Chen, L. T., Pricing and Production Lot-Size/Scheduling with Finite Capacity for a Deteriorating Item over a Finite Horizon, *Computers & Operations Research*, 32, 2005, pp. 2801-2819.
4. Chiu, S. W., Wang, S. L., and Chiu, Y. S. P., Determining the Optimal Run Time for EPQ Model with Scrap Rework, and Stochastic Breakdown, *European Journal of Operational Research*, 180, 2007, pp. 664 – 676.
5. Chung, C. J., Widyadana, G. A., and Wee, H. M., Economic Production Quantity Model for Deteriorating Inventory with Random Machine Unavailability and Shortage, *International Journal of Production Research*, 49(3), 2011, pp. 883 – 902.
6. El-Ferik, S., Economic Production Lot-Sizing for an Unreliable Machine under Imperfect Age-Based Maintenance Policy, *European Journal of Operational Research*, 186, 2008, pp. 150 – 163.
7. Giri, B. C., and Dohi, T., Computational Aspects of an Extended EMQ Model with Variable Production Rate, *Computers & Operations Research*, 32(12), 2005, pp. 3143 – 3161.
8. Lin, G. C., and Gong, D. C., On a Production-Inventory Model for Deteriorating Items Subject to an Imperfect Process, *Journal of the Chinese Institute of Industrial Engineers*, 24, 2007, pp. 319 – 326.
9. Misra, R. B., Optimal Production Lot-Size Inventory Model for Deteriorating Inventory, *International Journal of Production Research* 15, 1975, pp.495 – 505.
10. Mondal, S., and Maiti, M., Multi-item fuzzy EOQ Models using Genetic Algorithm, *Computers & Industrial Engineering*, 44, 2001, pp. 105 – 117.
11. Pasandideh, S. H. R., and Niaki, S. T. A., A Genetic Algorithm Approach to Optimize a Multi-Products EPQ Model with Discrete Delivery Orders and Constrained Space, *Applied Mathematics and Computations*, 195, 2008, pp. 506-514.
12. Pourakbar, M., Faharani, R. Z., and Asgari N. A., Joint Economic Lot-Size Model for Integrated Supply Network using Genetic Algorithm. *Applied Mathematics and Computation*, 189, 2007, pp. 583-596.
13. Teng, J. T., and Chang, C. T., Economic Production Quantity Models for Deteriorating Items with Price- and Stock-Dependent Demand, *Computers & Operations Research*, 32, 2005, pp. 297-308.
14. Tsao, Y. C., and Sheen, G. J., Dynamic Pricing, Promotion, and Replenishment Policies for a Deteriorating Item under Permissible Delay in Payments, *Computers & Operations Research*, 35, 2008, pp. 3562-3580.
15. Wee, H. M., Economic Production Lot Size Model for Deteriorating Items with Partial Back-ordering, *Computers & Industrial Engineering*, 24, 1993, pp. 449-458.
16. Wee H. M., and Law, S. T., Replenishment and Pricing Policy for Deteriorating Items Taking into Account the Time-Value of Money, *International Journal of Production Economics*, 71, 2001, pp. 213 – 220.
17. Widyadana, G. A., and Wee, H. M., A Replenishment Policy for Item with Price Dependent Demand and Deteriorating under Mark-down Policy, *Jurnal Teknik Industri*, 9(2), 2007, pp. 75-84.
18. Yang, P. C., and Wee, H. M., A Collaborative Inventory System with Permissible Delay in Payment for Deteriorating Items, *Mathematical and Computer Modelling,* 43, 2006, pp. 209 – 221.